The physics of the Nautilus

Seven classroom cards based on Jules Verne's Twenty Thousand Leagues Under the Sea

In Part I, chapter XIII, "Some Figures," Captain Nemo explains to Aronnax, with calculations, how the *Nautilus* floats, dives, withstands the pressure, and maneuvers. Each card has a passage from the book, the concept with its formula, a scene to set up in the 3D simulator, and questions for the class. The answers are at the end, for the teacher. The quotations are translated from Verne's French original.

3D simulator
mjrovai.github.io/
Vinte-Mil-Leguas/
simulador/?lang=en

How to use the simulator

Open the address above on a computer and click **Physics** in the bottom bar. The left panel shows, live, the weight, the buoyancy, the net force, the pressure, the force on the planes, the screw thrust, and the power. The **Piloting** panel, on the right, controls the engine, the rudder, the inclined planes, and the ballast. The arrows drawn on the hull use these colors:

- Weight W
- Buoyancy B
- Net force R = B − W (×10)
- Force on the planes (×40)
- Screw thrust (×40)
- Water resistance (×40)

Like Verne, we count 1 m^3 of water as 1 t in the weight and the buoyancy. In the ballast, 0 to 50% fills the main tanks (150.72 t) and 50 to 100% the supplementary ones (100 t). The pressure uses seawater (1,025 kg/m^3) and g = 9.8 m/s^2. The simulator moves the ship in a simplified way and with a compressed depth scale; the figures in the Physics panel follow the book's calculations, for the hull by the text. The quotations from Verne are translated here from the French original.

1 · Floating: Archimedes

> *"I wanted it, when balanced in the water, to be nine-tenths submerged and to stand out by one-tenth only."*
> Part I, ch. XIII

A body in water receives an upward push equal to the weight of the water it displaces: $\mathbf{B} = \rho \cdot \mathbf{V}_{sub} \cdot \mathbf{g}$. When it floats, the buoyancy balances the weight, **B = W**. The *Nautilus* weighs 1,356.48 t, so it sinks until it displaces 1,356.48 m^3 of water: nine-tenths of the hull's 1,507.2 m^3.

Put another way: the ship's average density, 1,356.48 ÷ 1,507.2 = 0.90 t/m^3, is 90% of the density of water. That is why 90% of the hull is underwater.

In the simulator

Open the **Physics** panel and click **Try it: the *Nautilus* afloat**. The tanks are empty. In Piloting, raise the ballast slowly to 50% and watch the submerged part go from 90% to almost 100%, with B always equal to W.

For the class

1 Nemo counted 1 m^3 of water as 1 t, which is fresh water. In seawater (1.025 t/m^3), how much of the hull would be out of the water?

2 · Diving: the ballast

> *"If I fill them with water, the boat, then displacing fifteen hundred and seven tons, or weighing them, will be completely submerged."*
> Part I, ch. XIII

With the whole hull underwater, the displaced volume no longer changes, and the buoyancy stays constant: **B = 1,507.2 t**. What changes is the weight, with the water that enters the tanks. The net force **R = B − W** decides: R > 0, the ship rises; R = 0, it stays where it is; R < 0, it sinks.

In terms of density: the ship rises if it is less dense than water and sinks if it is denser.

In the simulator

Open the **Physics** panel and click **Try it: stopped underwater**. The engine is stopped and the ballast is at 50%, the balance point. Move the ballast a little up or down and watch the yellow net-force arrow.

For the class

2 Why is it so hard to hold the *Nautilus* still at one depth using only the ballast?

3 · Going deep: pressure

> *"One atmosphere for every thirty feet of water, that is, about one kilogram per square centimeter."*
> Part I, ch. XIII

Pressure grows with depth: $\mathbf{p = p_0 + \rho \cdot g \cdot h}$. In seawater, every 10 m adds almost 1 atm (1,025 × 9.8 × 10 ≈ 100,000 Pa). Thirty French feet are about 9.7 m: Aronnax's rule is right.

The force on a surface is $\mathbf{F = \Delta p \cdot A}$, where Δp is the difference between the water pressure outside and the air pressure inside.

In the simulator

Open the **Physics** panel and click **Try it: dive to 1,000 m**. The autopilot takes the *Nautilus* down. Follow the pressure and the force on the salon window. If the sea floor is higher up, it settles there first.

For the class

3 At 1,000 m, what force does the water exert on a salon window? In this model, it is a 3.0 m × 1.64 m ellipse, with an area of ≈ 3.9 m².

4 · Maneuvering: the planes and the rudder

> *"If they are tilted, the Nautilus, according to the set of that tilt and under the thrust of its screw, either sinks along a diagonal as drawn out as suits me, or rises along that diagonal."*
> Part I, ch. XIII

The inclined planes and the rudder work like wings: the water flowing past pushes on them. The force grows with the square of the speed: $\mathbf{F = \tfrac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot C}$, where S is the area and C grows with the angle. When the ship is stopped, v = 0 and there is no force: the rudder does not turn the ship and the planes do not make it dive.

The same law holds for the water's resistance against the hull (gray arrow, above the deck), which the screw must overcome (lilac arrow, under the keel). At constant speed, the two are equal: **T = D**. When speeding up, the thrust is larger, and the difference is $\mathbf{m \cdot a}$. The engine power is $\mathbf{T \cdot v}$, so it grows with the cube of the speed.

In this model, S ≈ 16 m² (both planes) and C ≈ 0.04 per degree, up to a maximum of 1. For the resistance, the area is the hull's cross-section, ≈ 50 m², with C ≈ 0.2. The simulator speeds up much faster than a real ship, so the thrust jumps when you change the engine setting.

In the simulator

Open the **Physics** panel and click **Try it: going down with the planes**. The *Nautilus* is running at 10 knots with the planes at 15°, going down on a slant. Set the engine to 20 knots: the force on the planes and the resistance become four times larger. Before you reach the bottom, set the planes to rise; on the bottom, friction with the sand holds the ship.

For the class

4 At 15°, what speed do the planes need to produce the same force as the 150.72 t of water in the main tanks?

5 · Rising "like a balloon"

> *"I throw the screw out of gear, and the pressure of the water makes the Nautilus rise vertically, like a balloon that, filled with hydrogen, soars swiftly into the air."*
> Part I, ch. XIII

Nemo can sail with the hull a little lighter than water, with the net force $\mathbf{R = B - W}$ pointing up. What holds it down there are the inclined planes, with the screw providing speed. When the screw is disengaged, the speed drops, the force on the planes drops with its square, and only R is left, which is vertical. The ship rises straight up.

It is the same Archimedes as in the balloon: hydrogen (0.09 kg/m³) is much less dense than air (1.2 kg/m³), and the buoyancy beats the weight.

In the simulator

Open the **Physics** panel and click **Try it: disengage the screw**. The *Nautilus* is running at 15 knots, with the ballast at 45% (R ≈ 15 t upward) and the planes tilted down holding the hull. Click **Disengage screw** in Piloting, and watch the planes arrow shrink and the ship rise.

For the class

5 With R ≈ 15 t upward and a mass of 1,492 t, what is the acceleration at the start of the ascent? And up to what speed does it rise?

6 · Running fast: engine power

> *"A speed of fifty miles an hour."*
> Part I, ch. XII

To hold its speed, the screw must push as hard as the water resists: $\mathbf{T = D = \tfrac{1}{2} \cdot \rho \cdot v^2 \cdot A \cdot C}$. Power is force times speed: $\mathbf{P = T \cdot v}$. Since D grows with v^2, power grows with $\mathbf{v^3}$. Doubling the speed costs eight times the power.

Nemo says the power of his engines is "almost infinite." For comparison, Verne tells us in chapter I that the steamer *Scotia* made 13.4 knots with a thousand horsepower.

In the simulator

Open the **Physics** panel and click **Try it: 50 knots underwater**. Read the resistance and the power on the dashboard. Then bring the engine down to 25 knots and wait for the speed to settle.

For the class

6 If the speed drops from 50 to 25 knots, what happens to the resistance and to the power?

7 · Did Verne get it wrong?

> *"Its volume, fifteen hundred cubic meters and two-tenths."*
> Part I, ch. XIII

Verne made a point of putting exact figures in Nemo's mouth, and they can be checked pencil in hand, as Aronnax would. Three clues:

The volume. Nemo says the hull has 1,500.2 m^3 and floats nine-tenths submerged, 1,356.48 m^3. But nine-tenths of 1,500.2 is 1,350.18. Do the math with 1,507.2.

The screw. In chapter XII, it has a 7.5 m pitch and "can make up to a hundred and twenty turns per second," to reach fifty miles an hour. The pitch is how far the screw advances with each turn, like a bolt.

The shape. The measurements in the text and the drawing in the engravings do not give the same hull. The **Hull** button shows both shapes.

In the simulator

Open the **Physics** panel and click **Try it: switch the hull shape (the page reloads)**.

For the class

7.1 What is wrong with the volume?

7.2 With 120 turns per second and a 7.5 m pitch, how fast would the *Nautilus* go?

Answers

For the teacher.

1 *Nemo counted 1 m^3 of water as 1 t, which is fresh water. In seawater (1.025 t/m^3), how much of the hull would be out of the water?*
The submerged volume drops to 1,356.48 ÷ 1.025 ≈ 1,323 m^3, or 87.8% of the hull. About 12% would be out of the water, not 10%. Strictly speaking, Nemo's figures hold for a river or a lake.

2 *Why is it so hard to hold the Nautilus still at one depth using only the ballast?*
Because the balance is neutral. With R = 0 it stays where it is, but any ton more or less makes it sink or rise without stopping, since the buoyancy does not change with depth. That is why Nemo steers the ship with the inclined planes and the screw (card 4) and uses the ballast for the large adjustments. In the same chapter, he also notes that water compresses a little at depth, and that more weight is needed to make up for it.

3 *At 1,000 m, what force does the water exert on a salon window? In this model, it is a 3.0 m × 1.64 m ellipse, with an area of ≈ 3.9 m^2.*
Δp = 1,025 × 9.8 × 1,000 ≈ 10 million Pa. F = 10,000,000 × 3.9 ≈ 39 MN, the weight of about 4,000 t: almost three *Nautilus*es. Verne's answer is glass 21 cm thick at the center.

4 *At 15°, what speed do the planes need to produce the same force as the 150.72 t of water in the main tanks?*
F = 150.72 × 1,000 × 9.8 ≈ 1.48 MN. $v^2 = 2F \div (\rho \cdot S \cdot C) = 2 \times 1{,}480{,}000 \div (1{,}025 \times 16 \times 0.6) \approx 300$, so v ≈ 17 m/s, about 34 knots. The planes are for steering and correcting; the heavy work belongs to the ballast.

5 *With R ≈ 15 t upward and a mass of 1,492 t, what is the acceleration at the start of the ascent? And up to what speed does it rise?*
$a = F \div m = (15{,}000 \times 9.8) \div 1{,}492{,}000 \approx 0.1$ m/s^2, about 1% of g. The speed grows until the water's resistance equals R. Rising sideways, the area is the top view of the hull, ≈ 70 m × 8 m = 560 m^2, with C ≈ 1: $v = \sqrt{(2F \div (\rho \cdot A \cdot C))} = \sqrt{(2 \times 147{,}000 \div (1{,}025 \times 560))} \approx 0.7$ m/s, about 2.6 km/h.

6 *If the speed drops from 50 to 25 knots, what happens to the resistance and to the power?*
The resistance becomes 4 times smaller (from ≈ 348 t to ≈ 87 t), and the power 8 times smaller: from ≈ 88,000 kW (≈ 118,000 hp) to ≈ 11,000 kW (≈ 15,000 hp). At 50 knots, the *Nautilus* would need more than a hundred times the power of the *Scotia*.

7.1 *What is wrong with the volume?*
Nemo's calculations use 1,507.2 m^3, not 1,500.2. Nine-tenths of 1,507.2 is 1,356.48; the 150.72 t tanks make up the 1,507.2; and, further on, he says he will need to weigh 1,513.77 t instead of 1,507.2 t. There is also a 0.1 t gap: the two parts of the hull's weight, 394.96 t and 961.62 t, add up to 1,356.58 t, not 1,356.48. Both slips are already in the French original. This panel uses 1,507.2 m^3, the figure in the calculations.

7.2 *With 120 turns per second and a 7.5 m pitch, how fast would the Nautilus go?*
7.5 m × 120 = 900 m/s, or about 1,750 knots, more than twice the speed of sound in air. For 50 knots (25.7 m/s), ≈ 3.4 turns per second would do, or ≈ 206 per minute. If Verne meant 120 turns *per minute*, the screw would give 15 m/s, close to 29 knots, not counting slip in the water.

Simulator and cards generated by Anthropic's Claude Opus 5.5 under the direction of Marcelo Rovai, and translated into English by the same model. MIT License. More at mjrovai.github.io/Vinte-Mil-Leguas/en/